The MATH PRACTITIONER

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The Math Practitioner is a publication of the Adult Numeracy Network (ANN). We encourage submissions of articles, activities, and other items of interest related to math for adult learners. Please direct all correspondence to Donna Curry, Editor, at donnac@gwi.net.

From the President

The main article in this issue of *The Math Practitioner* focuses on the benefits of working in groups in math class. Patricia Helmuth, a New York adult numeracy consultant and educator, describes the ANN-supported practitioner research project in which she looks at whether group work will help students develop the "habits of mind" described in the *College and Career Readiness Standards for Adult Education*. Working with her high school equivalency class, and with other numeracy teachers, Patricia observed students as they worked together to solve open-ended math problems, verbalizing their thinking and developing new ways to look at problem solving. Read her article to see what Patricia learned, what students had to say, and how their pre- and post-test scores compared.

Other articles in this issue include Dorothea Steinke's presentation of a new acronym for looking at order of operations and Rebecca Strom's description of the Minnesota Numeracy Initiative (MNI), an ongoing, statewide professional learning project. In its fifth year, MNI is a comprehensive effort aimed at improving learners' numeracy outcomes, strengthening math content knowledge and self-confidence for teachers, increasing awareness of effective instructional strategies, enhancing collegiality and professionalism, and expanding the collection of numeracy activities and lessons for practitioner use. What is your state doing for math professional development? Write to us at donnac@gwi.net and let us know!

The student pages in this issue can be used as opportunities for students to develop those "habits of mind" as Patricia suggests. Try one a week with your students and see how their problem-solving and group interaction improve.

Finally, in the last issue, I talked about the Bridge college transition program where I teach. Bridge started in October and we have a very diverse group of students, ranging from those who struggle with basic math operations to those with college-level algebra skills. I've ended up working with two students who were drowning in the larger group and whose math skills are pretty basic. Both of them have some number sense, especially around money, which really helps with their foundational understanding of fractions, decimals and per cents. As children, both were told that they were terrible at math, would never really need it for anything, and would never improve. We're doing a lot of work with benchmarks – 1/2, 1/4, 3/4, 1/10 - using manipulatives and everyday applications. Stay tuned. The other night C. said, "I'm actually good at this, aren't I?" and I said, "Yep. Very good at this."

Sally Waldron

Would a Student's Overall Math Proficiency and Problem Solving Skills Be Boosted by Group Work?

by Patricia Helmuth

Patricia Helmuth is an Adult Numeracy Consultant and Educator. She teaches a high school equivalency class for Sullivan County Board of Co-operative Educational Services Adult Program, where she is coordinating a program for students under the age of 21. She is a participant of the NYSED Common Core/TASC Mathematics Teacher Learning & Leadership Institute as a Master Teacher, providing support for HSE instructors in the Hudson Valley, New York. While conducting this research she acted as the Lead Adult Numeracy Teacher/Trainer for Hudson Valley Catskill Partnership, where she served as PD Coordinator of the HVCP Adult Numeracy Cohort and the HVCP College and Career Readiness Master Teacher Cohort.

INTRODUCTION

With the release of the College and Career Readiness Standards for Adult Education and the implementation of new high school equivalency exams, adult educators are pondering how to best prepare their students to meet the expectations. One of these expectations is that students will develop "habits of mind" as described in the CCR Standards for Mathematical Practice. These habits of mind evince themselves in a student's ability to solve math problems using a variety of strategies that have developed over time, as well as being able to describe and defend a solution to the problem. According to the *Components of Numeracy*, this need to develop a productive disposition has long been recognized in "a number of the adult numeracy frameworks," and has a direct impact on the ability of a student to learn mathematics (pg. 30, *The Components of Numeracy*).

The purpose of this research was to examine and document if group work, which would include using manipulatives and visual models, would serve to help adult students develop these habits of mind. In particular, I was interested in MP.3, ***Construct viable arguments and critique the reasoning of others*** (*College and Career Readiness Standards for Adult Education*, pg. 48). Would group work serve as a vehicle to help build this kind of confidence in an adult education student, who may have come into class with a low opinion of his or her math skills? Would a student's overall math proficiency and problem solving skills be boosted by group work?

METHODS

This research was conducted using surveys, student reflections, direct classroom observation, and comparison of pre- and post-test TABE scores. I invited members of the HVCP Adult Numeracy Cohort to participate in this research, and three members of this professional development group responded to the call. These instructors also invited me into their classrooms, where I was able to make observations about group work efficacy in classrooms different than my own. Thus, the data collected represents students who reside at therapeutic communities, attend adult education classes at a community site, or attend classes at a BOCES adult education center. All classes are open-enrollment.

GROUP ACTIVITIES

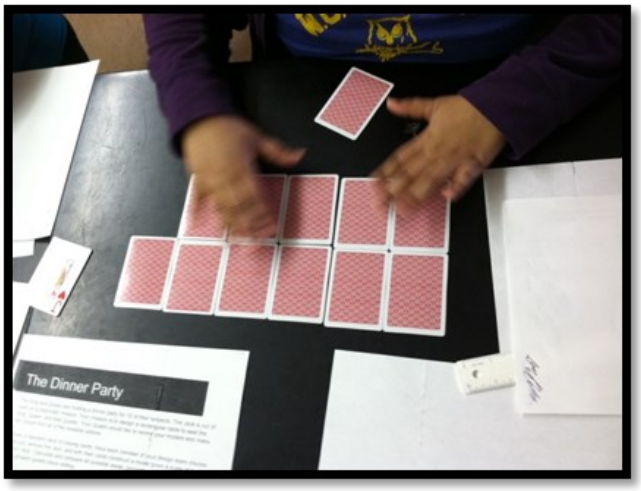
In my own classroom and in the classrooms I visited, group activities are designed so that students can work together to solve open-ended math problems. This means that when students are faced with a type of problem that they may not be familiar with, they don't need to rely on a traditional math algorithm to solve it. The solution pathway to the problem may wind up going in one direction with one group of students and in another direction with another group of students: both arriving at the same conclusion. The following are short descriptions of some of the activities in which these students participated.

Messy desks

In the picture on the left, my students are working with pattern blocks to explore fractional relationships. Using pattern blocks has proven to be a very productive activity for my students as they explore fractions, geometry, and algebra. It stimulates discussion between students as they move the pattern blocks around to solve problems. Additionally, it provides valuable information for the instructor, as he or she observes how students are reasoning, step-by-step. Understanding what students are thinking is the best first-step towards providing instruction at the point most needed by the students.

In the activity on the right, students are asked to arrange a deck of cards to model a banquet table for the Queen and King, who are having a dinner party for twelve. Students were directed to measure each card and given a scale to follow: one inch = one foot. They then arranged the cards in different ways until they were satisfied that each person at the dinner party would have enough room to eat. Students calculated area and perimeter with each arrangement, and there was meaningful discussion between the students as they drew conclusions about which would be the best. This activity enabled students to work on a project that required real-life numeracy skills including area, perimeter, and proportional reasoning. [The handout of this activity is on page 10.]

The Dinner Party



From the classroom of Larry Gotham
HSE Instructor for Orange-Ulster BOCES

Comment from Larry: "All of us (*members of the HVCP Adult Numeracy PDG*) are now engaging our students in a new classroom experience where manipulatives, concrete understanding, real world applications, and group discussion is encouraged, where mistakes and errors in thinking are accepted as a natural pathway to understanding.

Is it half?

From the classroom of Cynthia Hacker
HSE Instructor for Orange-Ulster BOCES

This activity required that students move through six different stations where they had to determine if the amount of an item that was placed at each station was exactly one -half, less than one-half, or more than one-half of the whole amount. Pictured is Station #6, where you see a number of candies on a plate that had been taken out of a package of candies. Students had to examine the Nutrition Facts label in order to determine if the amount of candies on the plate was 1/2, less than 1/2, or more than 1/2 of the entire package of candies. The label didn't directly state how many candies came in the package but provided enough information for the students to be able to figure it out. Students struggled a bit here, but didn't give up. I observed a group that eventually figured it out when they used the candies as manipulatives to establish serving size.

A comment from Cynthia: "So far my students have been positive, active, and enthusiastic about everything I have tried. They really like problem solving together, looking at math ...in a more visual way."

Benchmark fractions

This activity, utilizing real-life models that are placed around the room at various stations, represented benchmark fractions such as: 1/2, 1/3, 1/4, and 1/10. At the station pictured, a safety-pin was placed at a certain point on a pair of pants. What benchmark fraction would represent the distance between the waistline and the pin? I observed a lot of great discussions as I shadowed two students who moved from station to station and noticed that these students built confidence as they went along. These students were able to express their arguments in a very confident way, yet didn't seem to feel threatened if their theory was questioned and critiqued by the other party in their group. In the end, they came to a common conclusion.

From the classroom of Sherri Lattimer
HSE Instructor at Orange-Ulster BOCES

A comment from Sherri: "They *(the students)* realized that the manipulative activities were not as easy as they thought. The manipulatives, especially paper-folding, have instilled the concept, not just the process of solving fractions equations."

These kinds of math activities help students to realize that they can solve math problems even if they don't know traditional math algorithms. One student explained, "I could never understand or remember the steps [but now] "I understand how it connects to other things." Another student, after concluding that half-remembered and misapplied rules were constantly sending him off in the wrong direction, wrote this on his exit survey: "Forget the rules."

STUDENT REFLECTIONS

Student reflections on the activities provide convincing evidence that working in groups leads to opportunities for students to verbalize their thinking and to develop new ways to look at problem solving through the feedback of others in the group. Here are some of the comments:

"I learned benchmark fractions. I like doing hands on learning. I learn more that way."

"It helped me to visualize fractions better than I ever did before."

"This activity helps me when I'm working with groups of two or three and it helps me understand visually how fractions are used daily. Everyone thinks differently or has a different idea to work out mathematics subject."

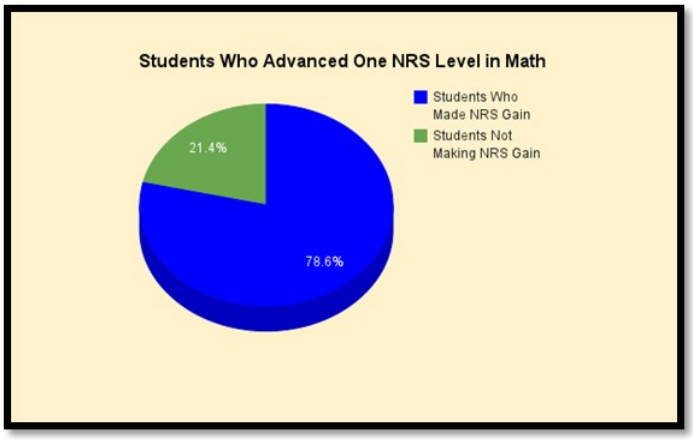
"Four eyes are better than two."

One student remarked on what stood out in his mind after participating in a group activity, "The way that we found the answer. We all had a different way."

A final comment from one student who gave this advice to new students on her exit survey: "When you do fractions and it's division, you take out the middle number and times the end ones first. If you get struck, draw a picture or use blocks because it helps." This comment underscores how exposure to group work strategies, that include manipulatives and visual models, can equip our students with problem solving skills that they can draw on when attempts to retrieve or apply memorized algorithms fail.

PRE AND POST-TEST SCORE COMPARISON

As the graph on the right indicates, 79% of the students who participated in group work had scores that went up one NRS level. Of the 21% that did not make a gain, there was either no post-test information available for the student or the score of the student showed slight improvement but not improved enough to count as an NRS gain.



CONCLUSIONS

A look at pre- and post-test scores of the students in this study suggests that group work may boost scores, as 79% of the students in this research achieved an NRS gain of at least one level in math. However, I stop short of pointing to this as conclusive evidence that group work *will* increase test scores. More research would be needed here; perhaps a controlled study which would compare test scores of a group of students who did not participate in group work with another group of students who did. However, what this research does establish is that taking the time in class to delve deeply into a math topic in a group situation does not seem to negatively impact student test scores. This is good news in the world of adult education, as we typically have our students for a limited amount of time.

Time is needed for a change in self-perception to grow and take firm root as is indicated by the exit survey where students rated themselves as only slightly improved in their math comfort level overall. While about half of the students surveyed rated their problem solving ability as improved, most students' self-reported comfort level in math showed minimal or no gain. This somewhat perplexes me, but suggests that more time

in class is needed for students to up their confidence level. Many, if not most, of our students come into our classes feeling as though they were beat up in math class in the past and it takes time to change their attitudes. It would be interesting to follow a group of students who are in a closed-enrollment class with a six month or more commitment, to see if their comfort level in math shows more improvement. This research was conducted in open-enrollment classes. We had to wait for new students to filter in and then give them intake surveys one at a time. Students also left or dropped out at different intervals, almost half not able to complete an exit survey.

Another finding of this research is that group activities can be successful in multi-level classrooms, with all skill levels benefiting from the activity and walking away with some new understanding. In my own classroom, I have had students whose intake scores on the TABE math test ranged from 3.0 - 9.0, yet all participated in the same activity. In some cases, I've given the more advanced student an additional difficulty to solve. In other cases, the more advanced student acts as mentor to the newer or lower ability student. Surprisingly, I've witnessed students with the lowest test score in his group, successfully work through and argue a solution to a hands-on activity with group members who were higher scoring but having difficulty with that particular activity. Thus, all students, regardless of their TABE math scores, have opportunities to "construct a viable argument and critique the reasoning of others."

This is not to say that all activities will work well with mixed ability groups. Sometimes, I paired students whose abilities were similar. At other times, through observation of former activities, I paired students that worked well together (regardless of their levels) and avoided pairing students that did not have a good working relationship. Finally, there are some students who still prefer to work independently and shut down if they are forced to pair with another student, so while I encouraged group participation, I did not require it. Most times, however, this avoidance of group work by a lone student was overcome eventually, as he or she observed the interaction between the students who were problem solving in pairs or groups.
For example, I had a student who was not participatory in class at all. It was his custom to find a seat as far away as he could from me and the other students. To my surprise, one day as I was forming groups for a volume activity, he stood up from his place far across the room, came forward, and asked, "What group am I in?" It turns out that previous work experience had given him some prior knowledge of volume and packaging. I watched as he turned his entire group around to his way of thinking. This was a turning point for him, and he made much progress after this.

On the intake survey, students rated the following statement: *I want to understand math better*. On a scale ranging from 1-10, with 1 signifying "Strongly Agree", 93% of the students chose #1 . Clearly, students come into class with the expectation that they will leave with a better understanding of mathematics. The results of this research imply that group work (which includes manipulatives, visual models, and contextualized situational problems) promotes understanding by fostering a learner-centered environment where all students are engaged and working together to explore the conceptual understanding of why a math rule works. Furthermore, group work creates a classroom situation where all students have the right and responsibility to express their thinking and argue a solution pathway, without fear of being judged negatively by their peers. Thus, not only can understanding come of it, but "habits of mind" that empower our students to become productive problem solvers can result. There is no single answer to what instructors and program managers can do that will enable our students to meet the expectations of the CCR Standards and new HSE exams. However, group work, as described in this practitioner research, is a step in that direction.

References:

Ginsburg, Lynda, Myrna Manly, Mary Jane Schmitt. (2006). *The Components of Numeracy*. http://www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf

GREMDAS, not PEMDAS

By Dorothea Steinke

Dorothea Steinke (B.A., Concordia University – Chicago; M. Music, University of Michigan) taught GED math and basic developmental math at Front Range Community College – Westminster, Colorado from 2009 through 2013. She holds the Colorado Adult Basic Education Authorization. She is currently involved with two grant projects for teaching math to young children using the math/music link.

There are two traps in the way that Order of Operations is taught in most texts. First, the acronym PEMDAS is incomplete. Second, the sequence in which the words of the acronym are said can lead students to use that sequence for all the operations.

The familiar textbook format lists the Order of Operations as:

1. parentheses
2. exponents
3. multiplication and division
4. addition and subtraction

What the PEMDAS acronym omits are: 1) the other grouping symbols, and 2) roots.
Using G to stand for grouping symbols allows for class discussion beyond parentheses ( ) to include brackets [ ] , braces { } , and the fraction line as a division sign that shows up in complex fractions.

Fig. 1 complex fraction

$$\frac{-(4+7)(6^2+12)}{(3+8)^2-19+12}$$

If they do not meet it sooner, students will encounter that fraction line grouping symbol when determining the slope of a line on the coordinate grid.

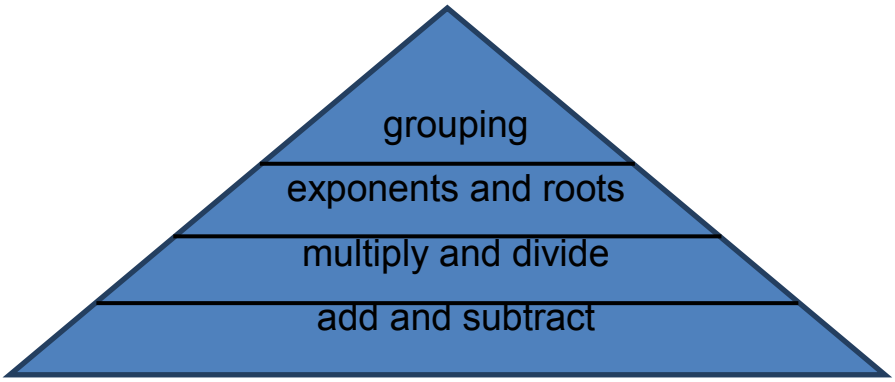
Fig. 2 slope formula

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

As for the second new letter, if students have been introduced to exponents, then including roots at the second level of the Order of Operations is no problem. Roots reverse the operation of exponentiation, just as division reverses the operation of multiplication.

Now that the Order of Operations model is more complete, let's put it in a visual that students can remember more easily. Using a simple picture activates different memory pathways than written words.

Fig. 3 Order of Operations Pyramid



Thinking of the steps as a pyramid allows students to go top down for the Order of Operations, as well as bottom up when solving equations. With the visual presentation, students also seem more able to get away from the word sequence (multiply, divide for example) and think of all the operations on a level as one step. This results in fewer errors in mathematical expressions that have a division sign before a multiplication sign, or a subtraction sign before an addition sign.

Using simple body motions can also enhance a student's memory. By tracing the outline of the pyramid with their hands, moving their hands wider and wider with each step as they say the words, students build a motor memory for the process.

Hear the words. See the graphic. Trace the pyramid in the air. Students now have a way to recall the process for the Order of Operations in all three learning modalities.

MN Loves Numeracy PD!

By Rebecca Strom

Rebecca Strom is a numeracy instructor for the Mankato Area ABE Program. She is also the Project Lead for the MNI (Minnesota Numeracy Initiative) .

In 2009, a Practitioner Survey for ABE teachers in MN revealed a need for professional development in the area of numeracy instruction. From that survey, the MN Numeracy Initiative (MNI) emerged. [For more information about MNI, go to http://atlasabe.org/professional/math-and-numeracy.]

I was very fortunate to be selected as a participant in its pilot year, and have been a part of it ever since. Each year, a cohort of 20-26 teachers have been selected across the state to participate in a year long training consisting of two full day face-to-face trainings, two online courses through World Education, weekly meetings with a partner, partner observation, and a final presentation at the end of the year. In addition to supporting our participants, MNI also offers support to all adult numeracy instructors across the state with regional and statewide conferences, including our own "Math Institute", where we have had ANN members help lead us with some of our Numeracy Trainings.

Out of our MNI training, many of our participants have continued to present what they've learned at our regional and statewide conferences, as well as traveling to share with other states and at COABE.

We definitely value the importance of continued professional development. This year, we were very excited to be able to offer each member of the new 2014-2015 cohort a membership to the Adult Numeracy Network, where they can be a part of a national community of numeracy instructors. ANN offers nationwide collaboration of ideas, support and leadership opportunities. We are very excited, also, to have won the registration to COABE's numeracy pre-conference, sponsored by ANN. We have had some of our MNI participants attend the past, myself included, and have come back with exciting new ideas that we have used in our classrooms and shared at conferences here in Minnesota. The MNI leadership team is very excited at the opportunity to send more participants again this year.

Thanks ANN!!

And ANN Loves MN!

To increase membership and celebrate 20 years of ANN, a membership drive was created in April 2014. The first state to recruit 20 new or renewing members from April by the deadline of December 31, 2014, would receive one paid registration to ANN's preconference at COABE to be held in Denver, Colorado in April 2015. Minnesota recruited 20 new members and one renewing member. Since the membership drive began, ANN has increased its membership by 68 new or renewing members. Thanks to all the states who actively participated and providing us with a strong membership!

The Dinner Party

The King and Queen are hosting a dinner party for 10 of their subjects. The Jack is out of town on a diplomatic mission. Your mission is to design a rectangular table to seat the King, Queen, and their guests. Your Queen would like to review your models and make her choice from all of the possible options.

Given a standard deck of playing cards, have each member of your design team choose one suit, remove the Jack, and with their cards construct a model given a scale of 1 inch=1 foot. Calculate and compare all possible areas, perimeters, and the available area of each guests' place setting.

Be prepared to demonstrate your results and give your Queen a professional recommendation as to which table would provide the best seating arrangement.

EXTENSION:

Given a standard deck of playing cards, how many different arrangements of area and perimeter are possible? How do your areas and perimeters compare? What is the largest possible area? The smallest? Be prepared to explain your reasoning and demonstrate your results.

From Larry Gotham, HSE Instructor for Orange-Ulster BOCES in NY

Group Problem-Solving Challenges

Four families each brought the same number of chairs to a block party. Three more chairs are needed to seat all 27 of the participants. How many chairs did each family bring?

It is the grand opening of a local supermarket. Every 5th customer will receive a coupon for a free turkey and every 7th customer will receive a coupon for a free half gallon of ice cream. If 400 customers come in on opening day, how many will get a free turkey and a free half gallon of ice cream?

You are pulling a raft up a river. You can make 15 miles a day. At night, while you are asleep, however, the current pushes the raft 3 miles back downstream. If you set out on Tuesday to get to a town 85 miles upstream, on which day will you arrive?

An example of consecutive odd numbers is 23, 25, 27, and 29. Find four consecutive odd numbers with a sum of 160.

From http://learn.fi.edu/school/math2/dec.html.

MEMBERSHIP APPLICATION FORM

Please help build membership in ANN. Pass this membership form along to another numeracy practitioner/ adult ed math teacher.

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Are you a member of the National Council of Teachers of Mathematics? Yes ☐ No ☐

Are you a member of COABE? Yes ☐ No ☐

Annual Dues for ANN Membership

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